Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name : Engineering Mathematics-I

Subject Code : 4TE01EMT1 **Branch: B.Tech(All)**

Semester : 1 **Date :22/03/2017** **Time :10:30 To 01:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q.1 Attempt the following questions: **(14)**

A) If the power of $x\,\&\,y$ both are even. then the curve is symmetrical about (1)

(a) X-axis (b) Y-axis (c) both X & Y axes (d) none of these

B) If $y=\sin^{-1}x$, then x equal to (1)

(a) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-......$ (b) $y-\frac{y^3}{3!}+\frac{y^5}{5!}-......$

(c) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}+......$ (d) none of these

C) Define Euler's Theorem. (1)

D) If $x=r\cos\theta\,\&\,y=r\sin\theta$, then $\frac{\partial(x,y)}{\partial(r,\theta)}=$ ______ (1)

(a) 0 (b) 1 (c) $\frac{1}{r}$ (d) r

E) $\lim_{x\to 0}\cos x=$ _____. (1)

$(a)0$ $(b)1$ $(c)\infty$ $(d)-1$

F) $e^{10\pi i}=$ (1)

(a) 0 (b) 1 (c) -1 (d) None of these

G) Find $\frac{dy}{dx}$ for $x^2+y^3=7xy.$ (1)

H) If the two tangents at the point are real & distinct, the double point is called (1)

(a) a node (b) a cusp (c) a conjugate point (d) None of these

I) The series $1-\frac{x^2}{2!}+\frac{x^4}{4!}-........$ represents expansion of (1)

(a) $\sin x$ (b) $\cos x$ (c) $\cosh x$ (d) $\sinh x$

J) The series $\sum \frac{1}{n}$ is (1)

(a) Convergent b) Divergent c) non-convergent d) a & b both

K) $\lim_{(x,y)\to(2,2)} \frac{3x+4y}{x+y} =$ ______. (1)

$(a)0$ $(b)1$ $(c)\frac{7}{2}$ $(d)\frac{3}{2}$

L) What is the argument of $z = 1+i$ (1)

$(a)-\frac{3\pi}{4}$ $(b)\frac{3\pi}{4}$ $(c)\frac{\pi}{4}$ $(d)-\frac{\pi}{4}$

.

M) $\lim_{x\to 0} \frac{x-\sin x}{x} =$ ______. (1)

(a) 0 b) 1 c) ∞ d)-1

N) If $J = \frac{\partial(u,v)}{\partial(x,y)}$ & $J' = \frac{\partial(x,y)}{\partial(u,v)}$ .then JJ'=……… (1)

(a) 1 (b) -1 (c) 0 (d) None of these

Attempt any four questions from Q-2 to Q-8

Q.2 Attempt all questions **(14)**

A) Trace the curve **(Cardioid)** $r = a(1+\cos\theta)$. **(07)**

B) Discuss the convergence of $\sum_{n=1}^{\infty}\left(\frac{\sqrt{n+1}-\sqrt{n}}{n}\right)$ **(04)**

C) Evaluate $\lim_{x\to 0}\left(\frac{1^x+2^x+3^x}{3}\right)^{\frac{1}{x}}$. **(03)**

Q.3 Attempt all questions **(14)**

A) Test the Convergence of the series $\sum_{n=1}^{\infty}\left(ne^{-n^2}\right)$. **(05)**

B) If $u = \ln\left(x^3+y^3+z^3-3xyz\right)$ then prove that **(05)**

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}.$$

C) Show that $\frac{(\cos 2\theta - i\sin 2\theta)^5(\cos 3\theta + i\sin 3\theta)^6}{(\cos 4\theta + i\sin 4\theta)^7(\cos\theta - i\sin\theta)^8} = \cos 12\theta - i\sin 12\theta.$ **(04)**

Q.4 Attempt all questions **(14)**

A) Test the series for absolute or conditional convergence **(05)**

$$1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\ldots\ldots$$

B) Find the Taylor's expansion of $\tan\left(x+\frac{\pi}{4}\right)$ in ascending powers of x up to x^4 & find approximate value of $\tan\left(43^\circ\right)$. **(05)**

C) Discuss the continuity of $f\left(x,y\right)=\begin{cases}\left(\dfrac{x^3-y^3}{x^2+y^2}\right); (x,y)\neq(0,0)\\ 0 \qquad ;(x,y)=(0,0)\end{cases}$ **(04)**

Q.5 Attempt all questions **(14)**

A) Trace the curve **(Cissoid of Diocle)** $y^2(2a-x)=x^3$. **(07)**

B) Find & plot all roots of $\sqrt[3]{8i}$. **(04)**

C) Evaluate: $\lim\limits_{x\to 0}\left(\dfrac{1}{x^2}-\dfrac{1}{\tan^2 x}\right)$. **(03)**

Q.6 Attempt all questions **(14)**

A) State Modified Euler's Theorem. If $u=\tan^{-1}\left(\dfrac{x^2+y^2}{x-y}\right)$, then show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\dfrac{1}{2}\sin 2u$ **(05)**

B) Prove that $\cos^8\theta=\dfrac{1}{128}\left[\cos 8\theta+8\cos 6\theta+28\cos 4\theta+56\cos 2\theta+35\right]$ **(05)**

C) Define $\log\left(x+iy\right)$ and Show that the set of $\log\left(i^2\right)$ is not same as the set of values of $2\log\left(i\right)$. **(04)**

Q.7 Attempt all questions **(14)**

A) Find maxima & minima of the Function $f\left(x,y\right)=x^2y-xy^2+4xy-4x^2-4y^2$. **(05)**

B) Obtain the range of convergence $\sum\limits_{n=1}^{\infty}\left(\dfrac{x^n}{2^n}\right), x>0$. **(05)**

C) Find $\dfrac{dy}{dx}$.if $y^{x^y}=\sin x$ **(04)**

Q.8 Attempt all questions **(14)**

A) Find the radius of convergence & interval of convergence of the series $\sum\limits_{n=0}^{\infty}\dfrac{(-3)^n x^n}{\sqrt{n+1}}$. **(05)**

B) Define Errors. Find the percentage error in calculating the area of an ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$,when error of 1% is made in measuring its major & find minor axes. **(05)**

C) If $x=r\sin\theta\cos\varphi, y=r\sin\theta\sin\varphi, z=r\cos\theta,$ *find* $\dfrac{\partial\left(x,y,z\right)}{\partial\left(r,\theta,\varphi\right)}$. **(04)**